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HEAT AND MASS TRANSFER OF CIRCULAR HOLLOW CYLINDERS UNDER BOUNDARY CONDITIONS OF THE SECOND KIND

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Abstract—The paper deals with a general solution of an internal heat and mass transfer problem for an infinite hollow cylinder under boundary conditions of the second kind.

NOMENCLATURE

- t, temperature, °C;
- u, moisture content, g moisture/g dry matter;
- r, radial co-ordinate, cm;
- R, bounding surface radius, cm;
- τ, time, sec;
- λ, thermal conductivity coefficient, cal/cm s deg C;
- c, specific heat capacity of moist body, cal/g deg C;
- γ_0, density of absolutely dry matter, g/cm^3;
- a, thermal diffusivity coefficient, cm^2/s $\left(= \frac{\lambda}{c\gamma_0}\right)$;
- k', moisture conductivity coefficient, cm^2/s;
- ρ, specific heat of evaporation, cal/g;
- δ, thermal gradient coefficient, 1/deg C;
- ϵ, coefficient of moisture internal evaporation;
- q, heat flow on surface, cal/cm^2 s;
- m', moisture flow on surface (drying intensity), g/cm^2 s;
- $J_n(x)$, $Y_n(x)$, Bessel functions of first and second type.

Dimensionless relations and criteria

$$k = \frac{R_2}{R_1} \quad Fo_1 = \frac{a\tau}{R_1^2}, \quad Fo_1' = \frac{k'\tau}{R_1^2},$$

$$Lu = \frac{k'}{a}, \quad Fe = \frac{\epsilon\rho\delta}{c};$$

$$Ki_1 = \frac{R_1 q_{01}}{\lambda t_0}, \quad Ki_1' = \frac{R_1 m_{01}'}{k'\gamma_0 u_0},$$

$$Ko' = \frac{\rho u_0}{c t_0}, \quad Pn = \frac{\delta t_0}{u_0}.$$

Subscripts

- 0, initial state;
- 1, internal cylindrical surface;
- 2, external cylindrical surface.

At present, analytical methods of solving internal heat and mass transfer problems are widely used for developing technological processes for drying materials. In many practically important cases boundary problem conditions may be given in the form of specific heat and moisture flows dependent on time, known, for example, on the basis of experimental data.

Problems of such a type for a sphere, plate and solid infinite cylinder have been solved by Prudnikov [1].

The present paper deals with a general solution of an internal heat and mass transfer problem for an infinite hollow cylinder.

The problem is as follows: An infinite hollow cylinder is given. The initial temperature and the moisture distribution through its section are functions of the radius. At $\tau > 0$ time dependences of specific heat and moisture flows on internal and external cylindrical surfaces are given.

One should determine temperature and moisture fields in a body at any time τ.

For the zonal calculation method of the process, the heat and mass transfer equations after Luikov [2] are:

Heat transfer differential equation:

$$\frac{\partial t}{\partial \tau} = a\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) + \frac{\epsilon \rho}{c}\frac{\partial u}{\partial \tau}, \quad (R_1 \leqslant r \leqslant R_2). \tag{1.1}$$

Mass transfer differential equation:

$$\frac{\partial u}{\partial \tau} = k'\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + k'\delta\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right). \tag{1.2}$$

Initial conditions:

$$t(r, 0) = \varphi(r), \quad u(r, 0) = \psi(r). \tag{2}$$

Boundary conditions:

$$\left.\begin{aligned}
&\frac{\partial t(R_2, \tau)}{\partial r} = \frac{1}{\lambda} q_2(\tau) \\
&\frac{\partial t(R_1, \tau)}{\partial r} = -\frac{1}{\lambda} q_1(\tau) \\
&\frac{\partial u(R_2, \tau)}{\partial r} + \delta\frac{\partial t(R_2, \tau)}{\partial r} = -\frac{m_1'(\tau)}{k'\gamma_0} \\
&\frac{\partial u(R_1, \tau)}{\partial r} + \delta\frac{\partial t(R_1, \tau)}{\partial r} = \frac{m_1'(\tau)}{k'\gamma_0}
\end{aligned}\right\} \tag{3}$$

Let us apply to the system (equations 1–3) the finite integral transformation determined by the expression:

$$\bar{f}_n = \int_{R_1}^{R_2} r f(r)\, W_0\left(\omega_n \frac{r}{R_1}\right) \mathrm{d}r \tag{4}$$

with the inversion formula:

$$f(r) = \frac{\int_{R_1}^{R_2} r f(r)\, \mathrm{d}r}{\int_{R_1}^{R_2} r\, \mathrm{d}r} + \frac{\mathrm{J}_1^2}{2R_1^2}\sum_{n=1}^{\infty}\frac{\omega_n^2 \mathrm{J}_1^2(k\omega_n)\, \bar{f}_n}{\mathrm{J}_1^2(\omega_n) - \mathrm{J}_1^2(k\omega_n)} W_0\left(\omega_n \frac{r}{R_1}\right), \tag{5}$$

where

$$W_0\left(\omega_n \frac{r}{R_1}\right) = \mathrm{Y}_1(\omega_n)\mathrm{J}_0\left(\omega_n \frac{r}{R_1}\right) - \mathrm{J}_1(\omega_n)\mathrm{Y}_0\left(\omega_n \frac{r}{R_1}\right), \tag{6}$$

ω_n is the root of an equation:

$$\mathrm{Y}_1(\omega_n)\mathrm{J}_1(k\omega_n) - \mathrm{J}_1(\omega_n)\mathrm{Y}_1(k\omega_n) = 0 \tag{7}$$

Thus, the system of ordinary differential equations is obtained:

$$\left.\begin{aligned} \frac{d\bar{t}_n}{d\tau} &= -\frac{\omega_n^2 a}{R_1^2}\bar{t}_n + \frac{\epsilon\rho}{c}\frac{d\bar{u}_n}{d\tau} + \frac{kR_1}{c\gamma_0}W_0(k\omega_n)q_2(\tau) + \frac{R_1}{c\gamma_0}W_0(\omega_n)q_1(\tau),^* \\ \frac{d\bar{u}_n}{d\tau} &= -\frac{\omega_n^2 k'}{R_1^2}\bar{u}_n - \frac{\omega_n^2 k'\delta}{R_1^2}\bar{t}_n - \frac{kR_1}{\gamma_0}W_0(k\omega_n)m_2'(\tau) - \frac{R_1}{\gamma_0}W_0(\omega_n)m_1'(\tau), \end{aligned}\right\} \quad (8)$$

under the initial conditions

$$\left.\begin{aligned} \bar{t}_n(0) &= \int_{R_1}^{R_2} r\varphi(r)W_0\left(\omega_n\frac{r}{R_1}\right)dr = \bar{\varphi}_n, \\ \bar{u}_n(0) &= \int_{R_1}^{R_2} r\psi(r)W_0\left(\omega_n\frac{r}{R_1}\right)dr = \bar{\psi}_n. \end{aligned}\right\} \quad (9)$$

After simple transformations the system (equations 8) is reduced to the form:

$$\left.\begin{aligned} \frac{d\bar{t}_n}{d\tau} &= a_{11}\bar{t}_n + a_{12}\bar{u}_n + f_1(\tau), \\ \frac{d\bar{u}_n}{d\tau} &= a_{21}\bar{t}_n + a_{22}\bar{u}_n + f_2(\tau), \end{aligned}\right\} \quad (8a)$$

where:

$$\left.\begin{aligned} a_{11} &= -\frac{\omega_n^2 k'}{R_1^2}\left(Fe + \frac{1}{Lu}\right), \\ a_{12} &= -\frac{\omega_n^2 k'}{R_1^2}\frac{\epsilon\rho}{c}, \\ a_{21} &= -\frac{\omega_n^2 k'}{R_1^2}\delta, \\ a_{22} &= -\frac{\omega_n^2 k'}{R_1^2}, \\ f_1(\tau) &= -\frac{2R_1}{\pi c\gamma_0}\frac{J_1(\omega_n)}{\omega_n J_1(k\omega_n)}q_2(\tau) - \frac{2R_1}{\pi c\gamma_0}\frac{1}{\omega_n}q_1(\tau) \\ &\quad + \frac{2R_1\epsilon\rho}{\pi c\gamma_0}\frac{J_1(\omega_n)}{\omega_n J_1(k\omega_n)}m_2'(\tau) + \frac{2R_1\epsilon\rho}{\pi c\gamma_0}\frac{1}{\omega_n}m_1'(\tau), \\ f_2(\tau) &= \frac{2R_1}{\pi\gamma_0}\frac{J_1(\omega_n)}{\omega_n J_1(k\omega_n)}m_2'(\tau) + \frac{2R_1}{\pi\gamma_0}\frac{1}{\omega_n}m_1'(\tau). \end{aligned}\right\} \quad (10)$$

* For future calculations it is taken into account that:

$$W_0(k\omega_n) = -\frac{2}{\pi k\omega_n}\frac{J_1(\omega_n)}{J_1(k\omega_n)},$$

$$W_0(\omega_n) = -\frac{2}{\pi\omega_n}.$$

The D'Alembert method is used for solving the linear system (equation 8) of non-uniform differential equations of the first order with constant coefficients under the initial conditions of equation (9).

As a result the system of two equations is obtained:

$$\bar{t}_n + \gamma_j \bar{u}_n = \exp\,[(a_{11} + \gamma_j a_{21})\tau]\ \{\bar{\varphi}_n + \gamma_j \bar{\psi}_n + \int_0^\tau [f_1(\tau) + \gamma_j f_2(\tau)] \exp\,[-(a_{11} + \gamma_j a_{21})\tau]\,d\tau\}, \quad (j = 1, 2) \tag{11}$$

where γ_j are the roots of an equation:

$$a_{12} + \gamma a_{22} = \gamma(a_{11} + \gamma a_{21}). \tag{12}$$

From (12) it follows:

$$\gamma_j = \frac{(a_{22} - a_{11}) + (-1)^j \sqrt{[(a_{22} - a_{11})^2 + 4a_{21}a_{12}]}}{2a_{21}}. \tag{13}$$

The discriminant of equation (12) is:

$$\mathscr{D} = -4a_{12}a_{21} - (a_{22} - a_{11})^2 < 0.$$

Hence the roots of equations (12) are differential and real.

Let us transform the radicant:

$$(a_{22} - a_{11})^2 + 4a_{12}a_{21} = (a_{22} + a_{11})^2 - 4(a_{22}a_{11} - a_{12}a_{21}) = \left(\frac{\omega_n^2 k'}{R_1^2}\right)^2 \left[\left(1 + Fe + \frac{1}{Lu}\right)^2 - \frac{4}{Lu}\right]$$

and designate:

$$\nu_j^2 = \frac{1}{2}\left\{\left(1 + Fe + \frac{1}{Lu} - \sqrt{\left[\left(1 + Fe + \frac{1}{Lu}\right)^2 - \frac{4}{Lu}\right]}\right\}. \tag{14}$$

The tables of values $\nu_{j(j=1,2)}$ for the criterion Lu from 0 to 100, and for the criterion Fe from 0 to 1, are obtainable [2].

It is easy to see that:

$$\gamma_j = \frac{1}{\delta}\left(\nu_j^2 - Fe - \frac{1}{Lu}\right). \tag{15}$$

Taking into account equations (10–15), the system (equation 11) may be written as follows:

$$\left.\begin{aligned} &\bar{t}_n + \frac{1}{\delta}\left(\nu_j^2 - Fe - \frac{1}{Lu}\right)\bar{u}_n = \exp\left(-\omega_n^2 \frac{k'\tau}{R_1^2}\nu_j^2\right) \times \left\{\bar{\varphi}_n + \frac{1}{\delta}\left(\nu_j^2 - Fe - \frac{1}{Lu}\right)\bar{\psi}_n\right. \\ &\left.+ \int_0^\tau \left[f_1(\tau) + \frac{1}{\delta}\left(\nu_j^2 - Fe - \frac{1}{Lu}\right) f_2(\tau)\right] \exp\left(\omega_n^2 \frac{k'\tau}{R_1^2}\nu_j^2\right) d\tau\right\}, \quad (j = 1, 2) \end{aligned}\right\} \tag{16}$$

$$\bar{u}_n = \frac{1}{\nu_2^2 - \nu_1^2}\,[P_{n2} \exp(-\omega_n^2 Fo_1' \nu_2^2) - P_{n1} \exp(-\omega_n^2 Fo_1' \nu_1^2)], \tag{17}$$

$$\bar{t}_n = -\frac{1}{\nu_2^2 - \nu_1^2}\,[\mathscr{L}_{n2} \exp(-\omega_n^2 Fo_1' \nu_2^2) - \mathscr{L}_{n1} \exp(-\omega_n^2 Fo_1' \nu_1^2)], \tag{18}$$

where:

$$\left.\begin{aligned}P_{nj} = \delta \int_{R_1}^{R_2} r\varphi(r) W_0\left(\omega_n \frac{r}{R_1}\right) \mathrm{d}r + \left(\nu_j^2 - Fe - \frac{1}{Lu}\right) \int_{R_1}^{R_2} r\psi(r) W_0\left(\omega_n \frac{r}{R_1}\right) \mathrm{d}r - \frac{2R_1}{\pi\gamma_0} \frac{1}{\omega_n} \\ \int_0^\tau \left\{\frac{\delta}{c} \frac{\mathrm{J}_1(\omega_n)}{\mathrm{J}_1(k\omega_n)} q_2(\tau) + \frac{\delta}{c} q_1(\tau) - \left(\nu_j^2 - \frac{1}{Lu}\right)\left[\frac{\mathrm{J}_1(\omega_n)}{\mathrm{J}_1(k\omega_n)} m_2'(\tau) + m_1'(\tau)\right]\right\} \\ \times \exp\left(\omega_n^2 \frac{k'\tau}{R_1^2} \nu_j^2\right) \mathrm{d}\tau, \quad (j = 1, 2),\end{aligned}\right\} \quad (19)$$

$$\left.\begin{aligned}\mathscr{L}_{nj} = \left(\nu_{j\pm1}^2{}^* - Fe - \frac{1}{Lu}\right) \int_{R_1}^{R_2} r\varphi(r) W_0\left(\omega_n \frac{r}{R_1}\right) \mathrm{d}r + \frac{1}{\delta}\left(\nu_{j\pm1}^2 - Fe - \frac{1}{Lu}\right)\left(\nu_j^2 - Fe - \frac{1}{Lu}\right) \\ \times \int_{R_1}^{R_2} r\psi(r) W_0\left(\omega_n \frac{r}{R_1}\right) \mathrm{d}r - \frac{2R_1}{\pi\gamma_0} \frac{1}{\omega_n}\left(\nu_{j\pm1}^2 - Fe - \frac{1}{Lu}\right) \int_0^\tau \left\{\frac{1}{c} \frac{\mathrm{J}_1(\omega_n)}{\mathrm{J}_1(k\omega_n)} q_2(\tau)\right. \\ \left. + \frac{1}{c} q_1(\tau) - \frac{1}{\delta}\left(\nu_j^2 - \frac{1}{Lu}\right)\left[\frac{\mathrm{J}_1(\omega_n)}{\mathrm{J}_1(k\omega_n)} m_2'(\tau) + m_1'(\tau)\right]\right\} \exp\left(\omega_n^2 \frac{k'\tau}{R_1^2} \nu_j^2\right) \mathrm{d}\tau.\end{aligned}\right\} \quad (20)$$

The first term of inversion formula, equation (5), is:

$$\frac{\int_{R_1}^{R_2} ru \, \mathrm{d}r}{\int_{R_1}^{R_2} r \, \mathrm{d}r} = \frac{2}{R_1^2(k^2 - 1)} \int_{R_1}^{R_2} r\psi(r) \, \mathrm{d}r - \frac{2}{R_1\gamma_0(k^2 - 1)} \int_0^\tau [km_2'(\tau) + m_1'(\tau)] \, \mathrm{d}r, \quad (21)$$

$$\left.\begin{aligned}\frac{\int_{R_1}^{R_2} rt \, \mathrm{d}r}{\int_{R_1}^{R_2} r \, \mathrm{d}r} = \frac{2}{R_1^2(k^2 - 1)} \int_{R_1}^{R_2} r\varphi(r) \, \mathrm{d}r + \frac{2}{R_1 c\gamma_0(k^2 - 1)} \int_0^\tau [kq_2(\tau) + q_1(\tau)] \, \mathrm{d}\tau \\ - \frac{2\epsilon\rho}{R_1 c\gamma_0(k^2 - 1)} \int_0^\tau [km_2'(\tau) + m_1'(\tau)] \, \mathrm{d}\tau\end{aligned}\right\} \quad (22)$$

From inversion formula, equation (5), on the basis of equations (17–20) the finite problem solution is obtained:

$$\left.\begin{aligned}u = \frac{2}{R_1^2(k^2 - 1)} \int_{R_1}^{R_2} r\psi(r) \, \mathrm{d}r - \frac{2}{R_1\gamma_0(k^2 - 1)} \int_0^\tau [km_2'(\tau) + m_1'(\tau)] \, \mathrm{d}\tau + \frac{\pi^2}{2R_1^2(\nu_2^2 - \nu_1^2)} \\ \times \sum_{n=1}^\infty \frac{\omega_n^2 \mathrm{J}_1^2(k\omega_n)}{\mathrm{J}_1^2(\omega_n) - \mathrm{J}_1^2(k\omega_n)} W_0\left(\omega_n \frac{r}{R_1}\right) \times [P_{n2} \exp(-\omega_n^2 Fo_1' \nu_2^2) - P_{n1} \exp(-\omega_n^2 Fo_1' \nu_1^2)],\end{aligned}\right\} \quad (23)$$

$$\left.\begin{aligned}t = \frac{2}{R_1^2(k^2 - 1)} \int_{R_1}^{R_2} r\varphi(r) \, \mathrm{d}r + \frac{2}{R_1 c\gamma_0(k^2 - 1)} \int_0^\tau [kq_2(\tau) + q_1(\tau)] \, \mathrm{d}\tau - \frac{2\epsilon\rho}{R_1 c\gamma_0(k^2 - 1)} \\ \int_0^\tau [km_2'(\tau) + m_1'(\tau)] \, \mathrm{d}\tau - \frac{\pi^2}{2R_1^2(\nu_2^2 - \nu_1^2)} \sum_{n=1}^\infty \frac{\omega_n^2 \mathrm{J}_1^2(k\omega_n)}{\mathrm{J}_1^2(\omega_n) - \mathrm{J}_1^2(k\omega_n)} W_0\left(\omega_n \frac{r}{R_1}\right) \\ \times [\mathscr{L}_{n2} \exp(-\omega_n^2 Fo_1' \nu_2^2) - \mathscr{L}_{n1} \exp(-\omega_n^2 Fo_1' \nu_1^2)].\end{aligned}\right\} \quad (24)$$

* At $j = 1$ $j \pm = 2$ is taken; at $j = 2$ $j \pm = 1$ is taken.

Let us assume that the specific heat and moisture flows on the internal and external cylinder surfaces, as well as the initial temperature and moisture distribution through its section, are constant*), i.e.

$$t(r,0)=\varphi(r)=t_0=\text{const.},\quad u(r,0)=\psi(r)=u_0=\text{const.},$$
$$q_2(\tau)=q_{02}=\text{const.},\quad q_1(\tau)=q_{01}=\text{const.},$$
$$m_2'(\tau)=m_{02}'=\text{const.},\quad m_1'(\tau)=m_{01}'=\text{const.}$$

Then

$$P_{nj}=-\frac{2R_1^2}{\pi\omega_n^2\nu_j^2}\,\frac{R_1}{k'\gamma_0\omega_n}\left\{\frac{\delta}{c}\,\frac{J_1(\omega_n)}{J_1(k\omega_n)}\,q_{02}+\frac{\delta}{c}\,q_{01}-\left(\nu_j^2-\frac{1}{Lu}\right)\left[\frac{J_1(\omega_n)}{J_1(k\omega_n)}\,m_{02}'+m_{01}'\right]\right\}\times[\exp(\omega_n^2Fo_1'\nu_j^2)-1], \tag{25}$$

$$\mathscr{L}_{nj}=\frac{1}{\delta}P_{nj} \tag{26}$$

Hence we obtain:

$$\begin{aligned}\frac{u_0-u}{u_0}&=\frac{2(km_{021}'+1)}{k^2-1}Ki_1'Fo_1'+\frac{Ki_1Pn}{Lu}\,\frac{Q_{021}(r)}{\nu_1^2\nu_2^2(k^2-1)}+\frac{Ki_1'}{Lu}\,\frac{M_{021}'(r)}{\nu_2^1\nu_2^2(k^2-1)}\\&-\frac{Ki_1Pn}{Lu}\,\frac{\pi}{(\nu_2^2-\nu_1^2)}+\sum_{n=1}^{\infty}\frac{[q_{021}J_1(\omega_n)+J_1(k\omega_n)]}{\omega_n[J_1^2(\omega_n)-J_1^2(k\omega_n)]}J_1(k\omega_n)W_0\left(\omega_n\frac{r}{R_1}\right)\\&\times\left[\frac{1}{\nu_2^2}\exp(-\omega_n^2Fo_1'\nu_2^2)-\frac{1}{\nu_1^2}\exp(-\omega_n^2Fo_1'\nu_1^2)\right]+Ki_1'\frac{\pi}{(\nu_2^2-\nu_1^2)}\\&\times\sum_{n=1}^{\infty}\frac{[m_{021}'J_1(\omega_n)+J_1(k\omega_n)]}{\omega_n[J_1^2(\omega_n)-J_1^2(k\omega_n)]}J_1(k\omega_n)\,W_0\left(\omega_n\frac{r}{R_1}\right)\left[\left(1-\frac{1}{\nu_2^2Lu}\right)\exp(-\omega_n^2Fo_1'\nu_2^2)\right.\\&\left.-\left(1-\frac{1}{\nu_1^2Lu}\right)\exp(-\omega_n^2Fo_1'\nu_1^2)\right],\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{t-t_0}{t_0}&=\frac{2(kq_{021}+1)}{k^2-1}Ki_1Fo_1-\frac{2(km_{021}'+1)}{k^2-1}\epsilon\,Ki_1'Ko'Fo_1'+\frac{Ki_1}{Lu}\,\frac{Q_{021}(r)}{\nu_1^2\nu_2^2(k^2-1)}\\&+\frac{Ki_1}{LuPn}\,\frac{M_{021}'(r)}{\nu_1^2\nu_2^2(k^2-1)}+\frac{Ki_1}{Lu}\,\frac{\pi}{(\nu_2^2-\nu_1^2)}\sum_{n=1}^{\infty}\frac{[q_{021}J_1(\omega_n)+J_1(k\omega_n)]}{\omega_n[J_1^2(\omega_n)-J_1^2(k\omega_n)]}J_1(k\omega_n)\,W_0\left(\omega_n\frac{r}{R_1}\right)\\&\times\left[\frac{1}{\nu_2^2}\exp(-\omega_n^2\,Fo_1'\nu_2^2)-\frac{1}{\nu_1^2}\exp(-\omega_n^2\,Fo_1'\nu_1^2)\right]+\frac{Ki_1'}{Pn}\,\frac{\pi}{(\nu_2^2-\nu_1^2)}\\&\times\sum_{n=1}^{\infty}\frac{[m'_{21}J_1(\omega_n)+J_1(k\omega_n)]}{\omega_n[J_1^2(\omega_n)-J_1^2(k\omega_n)]}J_1(k\omega_n)\,W_0\left(\omega_n\frac{r}{R_1}\right)\left[\left(1-\frac{1}{\nu_2^2Lu}\right)\exp(-\omega_n^2\,Fo_1'\nu_2^2)\right.\\&\left.-\left(1-\frac{1}{\nu_1^2Lu}\right)\exp(-\omega_n^2\,Fo_1'\nu_2^2)\right],\end{aligned} \tag{28}$$

* These conditions are fulfilled, for example, for radiation heating with a constant rate of drying.

where the following ratios:

$$m'_{021} = \frac{m'_{02}}{m'_{01}}, \quad q_{021} = \frac{q_{02}}{q_{01}}$$

are used, as well as it being taken into account that:

$$\left.\begin{aligned} -\pi \sum_{n=1}^{\infty} \frac{[q_{021}J_1(\omega_n) + J_1(k\omega_n)]}{\omega_n[J_1^2(\omega_n) - J_1^2(k\omega_n)]} J_1(k\omega_n)\, W_0\left(\omega_n \frac{r}{R_1}\right) = \frac{Q_{021}(r)}{k^2-1} \\ = \frac{1}{k^2-1}\left[\frac{kq_{021}+1}{2}\left(\frac{r^2}{R_1^2} - \frac{k^2+1}{2}\right) + k(q_{021}+k)\left(\frac{k^2 \ln k}{k^2-1} - \ln\frac{r}{R_1} - \frac{1}{2}\right)\right], \end{aligned}\right\} \quad (29)$$

$$\left.\begin{aligned} -\pi \sum_{n=1}^{\infty} \frac{[m'_{021}J_1(\omega_n) + J_1(k\omega_n)]}{\omega_n[J_1^2(\omega_n) - J_1^2(k\omega_n)]} J_1(k\omega_n)\, W_0\left(\omega_n \frac{r}{R_1}\right) = \frac{M'_{021}(r)}{k^2-1} \\ = \frac{1}{k^2-1}\left[\frac{km'_{021}+1}{2}\left(\frac{r^2}{R_1^2} - \frac{k^2+1}{2}\right) + k(m'_{021}+k)\left(\frac{k^2 \ln k}{k^2-1} - \ln\frac{r}{R_1} - \frac{1}{2}\right)\right]. \end{aligned}\right\} \quad (30)$$

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Аннотация—В работе получено общее решение внутренней задачи тепло- и массообмена неограниченного полого цилиндра при граничных условиях второго рода.

Zusammenfassung—Es wird eine allgemeine Lösung angegeben für das Problem des inneren Wärme- und Stoffübergangs bei einem unendlich langen Hohlzylinder mit der Randbedingung zweiter Art.